Gravity as a zero-point-fluctuation force

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Sakharov has proposed a suggestive model in which gravity is not a separately existing fundamental force, but rather an induced effect associated with zero-point fluctuations (ZPF's) of the vacuum, in much the same manner as the van der Waals and Casimir forces. In the spirit of this proposal we develop a point-particle–ZPF interaction model that accords with and fulfills this hypothesis. In the model gravitational mass and its associated gravitational effects are shown to derive in a fully self-consistent way from electromagnetic-ZPF-induced particle motion (*Zitterbewegung*). Because of its electromagnetic-ZPF underpinning, gravitational theory in this form constitutes an "already unified" theory.

I. INTRODUCTION

Gravitational theory, whether in its scalar Newtonian form or its tensor general-relativistic form, is recognized to be essentially phenomenological in nature. As such, it invites attempts at derivation from a more fundamental set of underlying assumptions, and six such attempts are outlined in the standard reference book *Gravitation*, by Misner, Thorne, and Wheeler (MTW).[1]

Of the six approaches presented in MTW, perhaps the most far-reaching in its implications for an underlying model is one due to Sakharov; namely, that gravitation is not a fundamental interaction at all, but rather an induced effect brought about by changes in the quantum-fluctuation energy of the vacuum when matter is present.[2,3] In this view the attractive gravitational force is more akin to the induced van der Waals and Casimir forces, than to the fundamental Coulomb force. Although speculative when first introduced by Sakharov in 1967, this hypothesis has led to a rich and ongoing literature on quantum-fluctuation-induced gravity that continues to be of interest. In this approach the presence of matter in the vacuum is taken to constitute a kind of set of boundaries as in a generalized Casimir effect, and the question of how quantum fluctuations of the vacuum under these circumstances can lead to an action and metric that reproduce Einstein gravity has been addressed from several viewpoints.[4] These are treated in some detail in a comprehensive review by Adler on gravity as a symmetry-breaking effect in quantum field theory.[5]

On the basis of heuristic and dimensional arguments along general relativistic lines, Sakharov argues that in a vacuum-fluctuation model for gravity the Newtonian gravitational constant G should be determined by an expression of the form

$$G \sim \frac{c^5}{\hbar \int_0^{\omega_c} \omega\, d\omega}, \quad \omega_c \sim \left[\frac{c^5}{\hbar G} \right]^{1/2}, \tag{1}$$

where ω_c corresponds to an effective Planck cutoff frequency of the vacuum zero-point-fluctuation (ZPF) spectrum.[6] In this approach, the small (but finite) value of the gravitational constant is an inverse reflection of the high (but not infinite) value of the high-frequency cutoff of the ZPF.

In this paper we explore the Sakharov viewpoint on the basis of a conceptually simple, classical model (but including ZPF) in which matter, in the form of charged point particles (partons), interacts with the ZPF of the vacuum electromagnetic field. As part of this development the model predicts (1) to be precisely of the form

$$G = \frac{\pi}{2} \frac{c^5}{\hbar \int_0^{\omega_c} \omega\, d\omega}, \quad \omega_c = \left[\frac{\pi c^5}{\hbar G} \right]^{1/2}. \tag{2}$$

In order to constitute a self-consistent, viable basis for gravitation, however, a first-order ZPF model for gravity must provide not only a basis for calculation of the gravitational constant G (shown to reflect the ZPF cutoff), but must also account for the genesis of the gravitational mass, and the attractive inverse-square-law force. In the particular version of the Sakharov hypothesis pursued here, the mass is shown to correspond to the kinetic energy of ZPF-induced internal particle (parton) motion (ZPF "jitter," or *Zitterbewegung)*, while the force is found to be of a long-range retarded van der Waals type, associated with the broad-spectrum ZPF radiation fields generated by that same *Zitterbewegung* motion.

To arrive at the above results, basically we simply assemble together in a straightforward fashion previously published results regarding ZPF models of van der Waals and related effects in flat space-time.[7] When this is done, one finds the leading term in the interaction potential, previously unexamined, to be Newton's law with no free parameters to be fixed. In such a fashion the identification of this term as gravitational emerges naturally from the concatenation of the previously published results. Yet further evidence for the correctness of this interpretation is provided in Sec. VI, where details of the pattern that emerges are presented in the context of a self-consistent coherent picture of the underlying dynamics of the gravitational force that conforms to the facts as we know them.

II. VACUUM ZPF ELECTROMAGNETIC FIELDS (REF. 8)

In the classical approach used here, point-particle–ZPF interactions are treated on the basis that charged point-mass particles interact with a background of random classical electromagnetic zero-point radiation with energy spectrum (as in the quantum-mechanical case)

$$\rho(\omega)d\omega=\left[\frac{\omega^2}{\pi^2c^3}\right]\left[\frac{\hbar\omega}{2}\right]d\omega=\frac{\hbar\omega^3}{2\pi^2c^3}d\omega\ , \tag{3}$$

where the first factor in parentheses corresponds to the density of normal modes, the second to an average energy $\frac{1}{2}\hbar\omega$ per mode. This treatment of quantum field-particle interactions on the basis of a classical ZPF constitutes an analysis technique known in the literature as stochastic electrodynamics (SED).[9] SED is a well-defined framework that has a long history of success in yielding precise quantitative agreement with full QED treatments of such topics as Casimir[10–12] and van der Waals forces,[13] topics directly related to the one pursued here. In this approach $\hbar$ appears in the above expression simply in the role of scale factor, without need of quantum interpretation, and all other appearances of $\hbar$ in the development can be traced back to its appearance in this expression.

The spectral energy density represented by (3) formally diverges as ω^3. It is generally assumed, however, that the spectrum is effectively cut off at a frequency roughly corresponding to the Planck frequency,

$$\omega_P=(c^5/\hbar G)^{1/2}\ .$$

As we shall see in the following sections, this assumption is supported by the line of development presented here.

As a first step toward developing the hypothesized underlying ZPF basis of the gravitational interaction, we compare the forms of the spectral distribution of the ZPF of the electromagnetic fields as seen from unaccelerated and accelerated frames of reference.

Of particular significance with regard to the spectral distribution in an unaccelerated frame, given by (3), is the fact of its Lorentz invariance, which derives specifically from the spectrum's cubic dependence on frequency. The cubic spectrum is unique in its property that delicate cancellations of Doppler shifts with velocity boosts leaves the spectrum Lorentz invariant.[14]

In an accelerated frame, on the other hand, the detailed balance of Doppler-shift cancellations is negated, with the result that the spectral distribution takes the form[15]

$$\rho(\omega)d\omega=\left[\frac{\omega^2}{\pi^2c^3}\right]\left[1+\left[\frac{a}{\omega c}\right]^2\right]\times\left[\frac{\hbar\omega}{2}+\frac{\hbar\omega}{\exp(2\pi c\omega/a)-1}\right]d\omega\ , \tag{4}$$

where a is the proper acceleration relative to a Lorentz frame. This expression was originally derived by Boyer[15] on the basis of the random classical ZPF background assumed here. The purpose was to derive a result, first obtained by Davies[16] and Unruh[17] within the context of quantum field theory, which showed that, apart from an additional density-of-states factor $[1+(a/\omega c)^2]$, the spectral distribution seen by an accelerating observer assumes a thermal (Planck) form if one makes the identification $T=\hbar a/2\pi ck$ (k is Boltzmann's constant; T, absolute temperature). In commenting on the additional density-of-states factor, Boyer points out that the additional contribution beyond the thermal (Planck) form is related to the space-time properties of an accelerating reference frame. This gives us a clue that, via the equivalence principle, this additional term can be related to the gravitational interaction.[18]

Of special interest here, therefore, is not the thermal term of interest in the original treatment, but rather the leading terms

$$\rho'(\omega)=\rho_0(\omega)+\Delta\rho'(\omega)=\frac{\hbar\omega^3}{2\pi^2c^3}+\frac{\hbar\omega a^2}{2\pi^2c^5}\ . \tag{5}$$

These indicate that an accelerated observer would see the background ZPF spectrum augmented by a term proportional to the square of the acceleration. Application of the principle of equivalence then indicates that the additional spectral contribution seen in a frame with acceleration **a** should also be seen in a nonaccelerated frame with local gravitational field **g** produced by a gravitational mass m_g. Setting $\mathbf{g}=-\mathbf{a}=-\hat{1}_r Gm_g/r^2$, we obtain

$$\Delta\rho'(\omega)=\frac{\hbar\omega}{2\pi^2c^5}\left[\frac{Gm_g}{r^2}\right]^2\ . \tag{6}$$

Thus the principle of equivalence predicts an additional contribution to ZPF energy by gravitational mass, a requirement that must be met in any ZPF-based theory of gravitation. Since this additional contribution of energy is electromagnetic in nature, we must ascribe to mass an appropriate electromagnetic-field-generating function, a point to which we return in Sec. IV.

III. *ZITTERBEWEGUNG* MODEL

As our basic point-particle–ZPF interaction model, we represent matter as a collection of charged point-mass particles (partons), in accordance with standard theory. In the development that follows it is not necessary to invoke the details of particular parton representations (e.g., families of fractionally charged quarks) beyond certain general concepts, such as the "asymptotic freedom" of partons to respond to the high-frequency components of the ZPF spectrum as essentially free particles. It is necessary to focus to the charged-parton level, however, in order to represent properly the essentially equal magnitudes of proton and neutron contributions to gravitational mass. This accounts for the fact that charged and neutral matter participate equally in the gravitational interaction, based on underlying charged-parton interactions.

We begin our discussion of particle-field interactions by examining the properties of a simple charged harmonic oscillator of natural frequency ω_0 (corresponding to a binding force that is linear in displacement from equilibrium), located at the origin and immersed in zero-point radiation. The (nonrelativistic) equation of motion for a particle of mass m_0 and charge q, including radiation damping, is given by

$$m_0\ddot{\mathbf{r}}+m_0\omega_0^2\mathbf{r}=\left[\frac{q^2}{6\pi\epsilon_0 c^3}\right]\dddot{\mathbf{r}}+q\mathbf{E}_{ZP}\ . \tag{7}$$

If we introduce the dipole moment, $\mathbf{p}=q\mathbf{r}$, and the damping constant,

$$\Gamma=q^2/6\pi\epsilon_0 m_0 c^3\ ,$$

we can write (7) in a form convenient for later discussion,

$$\ddot{\mathbf{p}}+\omega_0^2\mathbf{p}=\Gamma\dddot{\mathbf{p}}+6\pi\epsilon_0 c^3\Gamma\mathbf{E}_{ZP}\ . \tag{8}$$

We will also be particularly interested in the kinetic energy, $\mathcal{E}=\frac{1}{2}m_0\dot{\mathbf{r}}^2$, which becomes

$$\mathcal{E}=\frac{\dot{\mathbf{p}}^2}{12\pi\epsilon_0 c^3\Gamma}\ . \tag{9}$$

Once written in this form, the oscillator equation of motion (8) and energy equation (9) refer only to the global properties of the oscillator (dipole moment $\mathbf{p}$, natural frequency ω_0, and damping constant Γ), and do not involve individual mechanical properties of the oscillator such as charge or mass.

With regard to the ZPF fields, the vacuum is assumed to be filled with a random classical zero-point electromagnetic radiation whose Fourier composition underlies the spectrum given in (3). Written as a sum over plane waves, the random radiation, which is homogeneous, isotropic, and Lorentz invariant, can be expressed as

$$\mathbf{E}_{ZP}(\mathbf{r},t)=\mathrm{Re}\sum_{\sigma=1}^{2}\int d^3k\,\hat{\boldsymbol{\varepsilon}}\left[\frac{\hbar\omega}{8\pi^3\epsilon_0}\right]^{1/2}\times e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t+i\theta(\mathbf{k},\sigma)}\ , \tag{10}$$

$$\mathbf{H}_{ZP}(\mathbf{r},t)=\mathrm{Re}\sum_{\sigma=1}^{2}\int d^3k(\hat{\mathbf{k}}\times\hat{\boldsymbol{\varepsilon}})\left[\frac{\hbar\omega}{8\pi^3\mu_0}\right]^{1/2}\times e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t+i\theta(\mathbf{k},\sigma)}\ , \tag{11}$$

where $\sigma=1,2$ denote orthogonal polarizations, $\hat{\boldsymbol{\varepsilon}}$ and $\hat{\mathbf{k}}$ are orthogonal unit vectors in the direction of the electric field polarization and wave propagation vectors, respectively, $\theta(\mathbf{k},\sigma)$ are random phases distributed uniformly on the interval 0 to 2π (independently distributed for each $\mathbf{k},\sigma$), and $\omega=kc$.

It is at this point that we need to consider the correspondence between the above equations and the parton-ZPF interaction of interest. First, we treat the parton as a two-dimensional (rather than three-dimensional) oscillator, drawing on previous studies that model spin as the "internal" angular momentum associated with two-dimensional *Zitterbewegung* motion.[19] Second, because we are interested primarily in the particle's high-frequency *Zitterbewegung* response to the ZPF, whose spectral density increases as ω^3, we may to first order neglect the binding-force term involving ω_0 (asymptotic freedom as it relates to the ZPF). Finally, we also neglect the radiation-damping force in comparison to the inertial force and ZPF driving terms. (Alternatively, the restoring-force and radiation-damping terms can be carried through in the derivations that follow, and appropriate approximations introduced at the end, without change of result.)

To obtain the expectation value of kinetic energy of the *Zitterbewegung* motion given by (9), $\langle\mathcal{E}\rangle=\langle\dot{\mathbf{p}}^2\rangle/12\pi\epsilon_0 c^3\Gamma$, we follow a procedure due to Rueda.[20] Under the asymptotically-free-particle assumptions stated in the paragraph above, the x component of (8) takes the form (with $\hat{\mathbf{x}}$ a unit vector in the x direction)

$$\frac{d\dot{p}_x}{dt}=6\pi\epsilon_0 c^3\Gamma(E_{ZP})_x=6\pi\epsilon_0 c^3\Gamma\,\mathrm{Re}\left[\sum_{\sigma=1}^{2}\int d^3k(\hat{\boldsymbol{\varepsilon}}\cdot\hat{\mathbf{x}})\left[\frac{\hbar\omega}{8\pi^3\epsilon_0}\right]^{1/2}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t+i\theta(\mathbf{k},\sigma)}\right]\ . \tag{12}$$

Integrating once with respect to time, we obtain

$$\begin{aligned}\dot{p}_x&=6\pi\epsilon_0 c^3\Gamma\,\mathrm{Re}\left[\int_0^\tau dt\sum_{\sigma=1}^{2}\int d^3k(\hat{\boldsymbol{\varepsilon}}\cdot\hat{\mathbf{x}})\left[\frac{\hbar\omega}{8\pi^3\epsilon_0}\right]^{1/2}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t+i\theta(\mathbf{k},\sigma)}\right]\\&=6\pi\epsilon_0 c^3\Gamma\,\mathrm{Re}\left[\sum_{\sigma=1}^{2}\int d^3k(\hat{\boldsymbol{\varepsilon}}\cdot\hat{\mathbf{x}})\left[\frac{\hbar\omega}{8\pi^3\epsilon_0}\right]^{1/2}\left[\frac{e^{-i\omega\tau}-1}{-i\omega}\right]e^{i\mathbf{k}\cdot\mathbf{r}+i\theta(\mathbf{k},\sigma)}\right]\ .\end{aligned} \tag{13}$$

The expectation value $\langle\dot{p}_x^2\rangle$ then follows from

$$\langle\dot{p}_x^2\rangle=\tfrac{1}{2}\mathrm{Re}\Big\langle 36\pi^2\epsilon_0^2c^6\Gamma^2\sum_{\sigma=1}^{2}\sum_{\sigma'=1}^{2}\int d^3k\int d^3k'(\hat{\boldsymbol{\varepsilon}}\cdot\hat{\mathbf{x}})(\hat{\boldsymbol{\varepsilon}}'\cdot\hat{\mathbf{x}})\left[\frac{\hbar\omega}{8\pi^3\epsilon_0}\right]^{1/2}\left[\frac{\hbar\omega'}{8\pi^3\epsilon_0}\right]^{1/2}\left[\frac{e^{-i\omega\tau}-1}{-i\omega}\right]\times\left[\frac{e^{i\omega'\tau}-1}{i\omega'}\right]\exp[i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}+i\theta(\mathbf{k},\sigma)-i\theta(\mathbf{k}',\sigma')]\Big\rangle\ , \tag{14}$$

where use of the complex conjugates and the notation $\frac{1}{2}$Re stems from the use of exponential notation. Equation (14) can, however, be simplified to

$$\langle\dot{p}_x^2\rangle=36\pi^2\epsilon_0^2c^6\Gamma^2\sum_{\sigma=1}^{2}\int d^3k(\hat{\boldsymbol{\varepsilon}}\cdot\hat{\mathbf{x}})^2\frac{\hbar\omega}{8\pi^3\epsilon_0}\frac{1-\cos(\omega\tau)}{\omega^2}\ , \tag{15}$$

where averaging over random phases involves the use of

$$\langle \exp[i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}+i\theta(\mathbf{k},\sigma)-i\theta(\mathbf{k}',\sigma')]\rangle=\delta_{\sigma\sigma'}\delta^3(\mathbf{k}-\mathbf{k}') . \tag{16}$$

With $\int d^3k \rightarrow \int d\Omega_k \int dk\, k^2$, and the angular integration in k taking the form

$$\int d\Omega_k \left[\sum_{\sigma=1}^{2} (\hat{\boldsymbol{\varepsilon}}\cdot\hat{\mathbf{x}})^2\right] = \int d\Omega_k[1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})^2]=\tfrac{8}{3}\pi , \tag{17}$$

we can rewrite (15) (with a change of variables to $\omega=kc$) as

$$\begin{aligned}\langle \dot{p}_x^2\rangle &= 12\epsilon_0\hbar c^3\Gamma^2\int_0^{\omega_c} d\omega\,\omega[1-\cos(\omega\tau)]\\ &=6\epsilon_0\hbar c^3\Gamma^2\omega_c^2\left[1+\frac{2}{(\omega_c\tau)^2}[1-\cos(\omega_c\tau)-(\omega_c\tau)\sin(\omega_c\tau)]\right] ,\end{aligned} \tag{18}$$

where ω_c is the assumed cutoff frequency, to be determined later.[21]

For $\omega_c\tau \gg 1$, $\langle \dot{p}_x^2\rangle$ reaches the value

$$\langle \dot{p}_x^2\rangle=6\epsilon_0\hbar c^3\Gamma^2\omega_c^2 . \tag{19}$$

For the two-dimensional *Zitterbewegung* motion assumed,

$$\langle \dot{\mathbf{p}}^2\rangle=2\langle \dot{p}_x^2\rangle=12\epsilon_0\hbar c^3\Gamma^2\omega_c^2 , \tag{20}$$

which, when substituted into (9), yields

$$\langle \mathcal{E}\rangle=\frac{\Gamma\hbar\omega_c^2}{\pi} . \tag{21}$$

It is thus seen that the expectation value of the kinetic energy of parton *Zitterbewegung* motion reaches a finite magnitude, limited by the finite value of the (as yet undetermined) ZPF cutoff frequency. The energy calculated in this way is in the nature of the so-called "transverse self-energy" (in QED) of a particle in response to the electromagnetic zero-point fluctuations of the vacuum.[22] Since the energy associated with this *Zitterbewegung* motion is an internal particle energy, that is, not directly observable, we identify this energy as that corresponding to the rest-mass energy of the particle, m,

$$m=\frac{\langle \mathcal{E}\rangle}{c^2}=\frac{\Gamma\hbar\omega_c^2}{\pi c^2} . \tag{22}$$

As will be shown in Sec IV, $\omega_c=(\pi c^5/\hbar G)^{1/2}$, in which case (22) reduces to

$$m=\frac{\Gamma c^3}{G} . \tag{23}$$

In this view the particle mass m is of dynamical origin, originating in parton-motion response to the electromagnetic zero-point fluctuations of the vacuum. It is therefore simply a special case of the general proposition that the internal kinetic energy of a system contributes to the effective mass of that system.[23] This derivation of mass as an internal kinetic energy of motion is thus the first result derived from the *Zitterbewegung* model. As will be shown in later sections, it is this mass that is involved in the gravitational interaction.[24]

IV. *ZITTERBEWEGUNG* FIELDS

We turn our attention now to the fields generated by the ZPF-induced *Zitterbewegung* motion. Considering, say, the x component of motion, we find that an assumed $e^{-i\omega t}$ time dependence substituted into (8) yields for the magnitude of any particular frequency component

$$\tilde{p}_x(\omega)=-\frac{6\pi\epsilon_0 c^3\Gamma}{\omega^2}(\hat{\boldsymbol{\varepsilon}}\cdot\hat{\mathbf{x}})\tilde{E}_{\rm ZP}(\omega) , \tag{24}$$

where the overtilde designates the magnitude of a frequency component, and once again we have neglected the binding and radiation-damping forces. This expression can then be combined with the ZPF-field expressions (10) and (11), and the standard oscillating dipole formulas[25]

$$\mathbf{E}_d(\omega)=\text{Re}\left[\frac{1}{4\pi\epsilon_0}\tilde{p}e^{-i\omega t}\mathbf{G}\right] , \tag{25}$$

$$\mathbf{H}_d(\omega)=\text{Re}\left[\frac{c}{4\pi}\tilde{p}e^{-i\omega t}\mathbf{F}\right] , \tag{26}$$

to yield expressions for the dipole fields generated by the *Zitterbewegung* motion, viz.,

$$\mathbf{E}_d=-\text{Re}\left[\frac{1}{4\pi\epsilon_0}\sum_{\sigma=1}^{2}\int d^3k\left[\frac{6\pi\epsilon_0c^3\Gamma}{\omega^2}\right]\left[\frac{\hbar\omega}{8\pi^3\epsilon_0}\right]^{1/2}(\hat{\boldsymbol{\varepsilon}}\cdot\hat{\mathbf{x}})e^{-i\omega t+i\theta(\mathbf{k},\sigma)}\mathbf{G}\right] , \tag{27}$$

$$\mathbf{H}_d=-\text{Re}\left[\frac{c}{4\pi}\sum_{\sigma=1}^{2}\int d^3k\left[\frac{6\pi\epsilon_0c^3\Gamma}{\omega^2}\right]\left[\frac{\hbar\omega}{8\pi^3\epsilon_0}\right]^{1/2}(\hat{\boldsymbol{\varepsilon}}\cdot\hat{\mathbf{x}})e^{-i\omega t+i\theta(\mathbf{k},\sigma)}\mathbf{F}\right] , \tag{28}$$

where, with $\hat{\mathbf{r}}$ a unit vector in the direction joining the dipole to the field evaluation point,

$$\mathbf{G}=k^3e^{ikr}\left[(\hat{\mathbf{r}}\times\hat{\mathbf{x}})\times\hat{\mathbf{r}}\left[\frac{1}{kr}\right]+[3\hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\hat{\mathbf{x}})-\hat{\mathbf{x}}]\left[\frac{1}{(kr)^3}-\frac{i}{(kr)^2}\right]\right], \tag{29}$$

$$\mathbf{F}=k^3e^{ikr}(\hat{\mathbf{r}}\times\hat{\mathbf{x}})\left[\frac{1}{(kr)}+\frac{i}{(kr)^2}\right]. \tag{30}$$

The energy density in the dipole-field distribution can be calculated from (27) and (28) as

$$\begin{aligned}\Delta w_d &= \tfrac{1}{2}\epsilon_0\langle \mathbf{E}_d^2\rangle+\tfrac{1}{2}\mu_0\langle \mathbf{H}_d^2\rangle\\ &=\tfrac{1}{2}\,\mathrm{Re}\Big\langle\frac{\epsilon_0}{2}\left[\frac{1}{4\pi\epsilon_0}\right]^2\sum_{\sigma=1}^{2}\sum_{\sigma'=1}^{2}\int d^3k\int d^3k'\left[\frac{6\pi\epsilon_0c^3\Gamma}{\omega^2}\right]\left[\frac{6\pi\epsilon_0c^3\Gamma}{\omega'^2}\right]\left[\frac{\hbar\omega}{8\pi^3\epsilon_0}\right]^{1/2}\left[\frac{\hbar\omega'}{8\pi^3\epsilon_0}\right]^{1/2}\\ &\qquad\times(\hat{\boldsymbol{\varepsilon}}\cdot\hat{\mathbf{x}})(\hat{\boldsymbol{\varepsilon}}'\cdot\hat{\mathbf{x}})|\mathbf{G}|^2\exp[-i(\omega-\omega')t+i\theta(\mathbf{k},\sigma)-i\theta(\mathbf{k}',\sigma')]\Big\rangle\\ &\quad+\tfrac{1}{2}\,\mathrm{Re}\Big\langle\frac{\mu_0}{2}\left[\frac{c}{4\pi}\right]^2\sum_{\sigma=1}^{2}\sum_{\sigma'=1}^{2}\int d^3k\int d^3k'\left[\frac{6\pi\epsilon_0c^3\Gamma}{\omega^2}\right]\left[\frac{6\pi\epsilon_0c^3\Gamma}{\omega'^2}\right]\left[\frac{\hbar\omega}{8\pi^3\epsilon_0}\right]^{1/2}\left[\frac{\hbar\omega'}{8\pi^3\epsilon_0}\right]^{1/2}\\ &\qquad\times(\hat{\boldsymbol{\varepsilon}}\cdot\hat{\mathbf{x}})(\hat{\boldsymbol{\varepsilon}}'\cdot\hat{\mathbf{x}})|\mathbf{F}|^2\exp[-i(\omega-\omega')t+i\theta(\mathbf{k},\sigma)-i\theta(\mathbf{k}',\sigma')]\Big\rangle.\end{aligned} \tag{31}$$

This reduces to

$$\Delta w_d=\frac{3\hbar c^3\Gamma^2}{16\pi^2}\int_0^{\omega_c}\frac{d\omega}{\omega}(|\mathbf{F}|^2+|\mathbf{G}|^2), \tag{32}$$

where we have made use of (17), and have averaged over random phases by the use of

$$\langle\exp[-i(\omega-\omega')t+i\theta(\mathbf{k},\sigma)-i\theta(\mathbf{k}',\sigma')]\rangle=\delta_{\sigma\sigma'}\delta_{\omega\omega'}\delta^3(\mathbf{k}-\mathbf{k}'). \tag{33}$$

The part of the integrand in parentheses in (32) can be evaluated by means of (29) and (30), and yields

$$|\mathbf{F}|^2+|\mathbf{G}|^2=k^6\left[\frac{2\sin^2\psi}{(kr)^2}+\frac{4\cos^2\psi}{(kr)^4}+\frac{4\cos^2\psi+\sin^2\psi}{(kr)^6}\right], \tag{34}$$

where ψ is the angle measured from the dipole-motion unit vector $\hat{\mathbf{x}}$ to the evaluation-point unit vector $\hat{\mathbf{r}}$. The first term proportional to $1/r^2$ constitutes the radiation field associated with the ZPF-driven dipole. As shown previously by Boyer,[26] this radiation just replaces that being absorbed from the background, on a detailed-balance basis with regard to both frequency and angular distribution, and therefore does not result in an incremental change to the background. Of the two remaining (induction) field terms, the $1/r^4$ term predominates at large distances, and is therefore the one of interest here. Designating the term of interest by a prime, we have

$$\Delta w_d'=\frac{3\hbar c\Gamma^2\cos^2\psi}{4\pi^2r^4}\int_0^{\omega_c}\omega\,d\omega. \tag{35}$$

This expression, obtained on the basis of considering a single (x) component of motion, must be doubled to take into account the contributions of the two (independent) degrees of freedom in the model. This leads then to an overall spectral density

$$\Delta\rho_d'(\omega)=\frac{3\hbar c\Gamma^2\omega\cos^2\psi}{2\pi^2r^4}. \tag{36}$$

Since in the final analysis we are interested in the net contribution of a large collection of randomly oriented individual particle motions, we average over the solid angle to obtain

$$\overline{\Delta\rho_d'(\omega)}=\frac{1}{4\pi r^2}\int_0^{\pi}\Delta\rho_d'2\pi r^2\sin\psi\,d\psi=\frac{\hbar c\Gamma^2\omega}{2\pi^2r^4}. \tag{37}$$

Since according to (22) there is a relationship between Γ and the particle mass m for ZPF-driven *Zitterbewegung* motion, (37) can also be written

$$\overline{\Delta\rho_d'(\omega)}=\frac{m^2c^5\omega}{2\hbar\omega_c^4r^4}. \tag{38}$$

Zitterbewegung motion therefore leads to the generation of an electromagnetic field distribution in proximity to the mass that is proportional to frequency times mass squared, divided by r^4, and upon detailed examination is found to be half electric, half magnetic. According to (6), moreover, a field of just this form is required by the principle of equivalence. Under the assumption that the gravitational and rest masses are identical ($m_g=m$), (6) and (38) can be equated to obtain the cutoff frequency

$$\omega_c=\left[\frac{\pi c^5}{\hbar G}\right]^{1/2} \tag{39}$$

which satisfies the Sakharov condition (1). In terms of the cutoff frequency ω_c, (39) can be inverted to yield the gravitational constant G in the form of the second (nonindependent) Sakharov condition, namely,

$$G=\frac{\pi c^5}{\hbar\omega_c^2}=\frac{\pi}{2}\frac{c^5}{\hbar\int_0^{\omega_c}\omega\,d\omega}. \tag{40}$$

We see therefore that the principle of equivalence requires a certain modification of the ZPF background by gravitational mass, and that the *Zitterbewegung* model of mass implies a similar modification. Furthermore, the compatibility of the two conditions on a precise quantitative basis requires only the equivalence of gravitational and rest masses, and a limiting cutoff frequency for the ZPF background on the order of the Planck frequency. This specification of the cutoff frequency ω_c, and its relationship to the gravitational constant G, is thus the second result derived from the *Zitterbewegung* model. Thus we have a derivation that yields the relationships postulated to exist in dynamical scale-invariance-breaking models of gravity as a symmetry-breaking effect.[6]

V. GRAVITATIONAL FORCE

The derivations in Secs. III and IV dealt essentially with the characteristics of single masses. In this section we investigate the interaction *between* two such masses.

The starting point is equations of the form (8), written for two masses, but modified to take into account the fact that each mass experiences not only the background ZPF field, but also the ZPF-driven dipole field of the other mass. The procedure followed here is precisely that developed by Boyer for the derivation of the retarded van der Waals forces at all distances between a pair of polarizable particles.[27] Therefore we need only outline the procedure as it applies to our case.

Two masses A and B (taken here to be equal for ease of discussion) are assumed to be located at positions $\mathbf{r}_A$ and $\mathbf{r}_B$, respectively, with B located a distance R from A, along the positive z axis of a coordinate system centered at A. The equations of motion, generalized from (8), take the form.

$$\ddot{\mathbf{p}}_A+\omega_0^2\mathbf{p}_A=\Gamma\dddot{\mathbf{p}}_A+6\pi\epsilon_0c^3\Gamma\mathbf{E}_{\mathrm{ZP}}(\mathbf{r}_A,t)+6\pi\epsilon_0c^3\Gamma\mathbf{E}_{dB}(\mathbf{r}_A,t)\ , \tag{41}$$

$$\ddot{\mathbf{p}}_B+\omega_0^2\mathbf{p}_B=\Gamma\dddot{\mathbf{p}}_B+6\pi\epsilon_0c^3\Gamma\mathbf{E}_{\mathrm{ZP}}(\mathbf{r}_B,t)+6\pi\epsilon_0c^3\Gamma\mathbf{E}_{dA}(\mathbf{r}_B,t)\ , \tag{42}$$

where $\mathbf{E}_{dB}(\mathbf{r}_A,t)$ is the dipole electric field at the position of particle A, due to the motion of particle B, and so forth.

As in the derivations of previous sections, an assumed $e^{-i\omega t}$ time dependence yields (for the magnitude of any particular frequency component) equations of the form

$$D\tilde{p}_{Ax}(\omega)+\eta_x\tilde{p}_{Bx}(\omega)=6\pi\epsilon_0c^3\Gamma\tilde{E}_{\mathrm{ZP}x}(\omega,\mathbf{r}_A)\ , \tag{43}$$

$$D\tilde{p}_{Bx}(\omega)+\eta_x\tilde{p}_{Ax}(\omega)=6\pi\epsilon_0c^3\Gamma\tilde{E}_{\mathrm{ZP}x}(\omega,\mathbf{r}_B)\ , \tag{44}$$

and similar equations for the y and z components, where

$$D=-\omega^2+\omega_0^2-i\Gamma\omega^3\ , \tag{45}$$

$$\eta_x=\eta_y=-\tfrac{3}{2}\Gamma\omega^3e^{ikR}\left[\frac{1}{(kR)}+\frac{i}{(kR)^2}-\frac{1}{(kR)^3}\right]\ , \tag{46}$$

$$\eta_z=-\tfrac{3}{2}\Gamma\omega^3 2e^{ikR}\left[-\frac{i}{(kR)^2}+\frac{1}{(kR)^3}\right]\ . \tag{47}$$

The latter expressions for η_x, etc., are derived for the geometry under consideration from the dipole expressions (25) and (29).

Consistent with the assumptions of previous sections, we neglect the binding and radiation-damping terms [terms in ω_0^2 and Γ in (45)]. Furthermore, on the scale of interest in gravitation (distances large compared with the wavelengths of the predominant *Zitterbewegung* frequencies), in calculating the force we retain only the radiation field term $\propto 1/kR$. With these assumptions, (45)–(47) become

$$D^{\mathrm{rad}}=-\omega^2\ , \tag{48}$$

$$\eta_x^{\mathrm{rad}}=\eta_y^{\mathrm{rad}}=-\tfrac{3}{2}\Gamma\omega^3\frac{e^{ikR}}{kR},\quad \eta_z^{\mathrm{rad}}=0\ . \tag{49}$$

Solutions for $\tilde{p}_{Ax}(\omega)$ and $\tilde{p}_{Bx}(\omega)$ are obtained straightforwardly[28] from (43) and (44), subject to the conditions (48) and (49). It is then necessary to construct from these solutions the appropriate (van der Waals) forces between the pair of particles A and B. From classical theory the force on a dipole is given by

$$\mathbf{F}=(\mathbf{p}\cdot\nabla)\mathbf{E}+\frac{d\mathbf{p}}{dt}\times\mathbf{B}\ . \tag{50}$$

Following Boyer,[27] we note that the time-averaged force can be transformed as

$$\begin{aligned}\langle\mathbf{F}\rangle&=\langle(\mathbf{p}\cdot\nabla)\mathbf{E}\rangle+\left\langle\frac{d\mathbf{p}}{dt}\times\mathbf{B}\right\rangle\\&=\langle(\mathbf{p}\cdot\nabla)\mathbf{E}\rangle+\langle\mathbf{p}\times(\nabla\times\mathbf{E})\rangle\ ,\end{aligned} \tag{51}$$

where the transformation of the second term

$$\left\langle\frac{d\mathbf{p}}{dt}\times\mathbf{B}\right\rangle=\frac{1}{T}\int_0^T\frac{d\mathbf{p}}{dt}\times\mathbf{B}\,dt \tag{52}$$

follows from an integration by parts and substitution from Maxwell's equations, noting that the end-point terms do not contribute to the time average. Manipulation of the vector operators in (51) then leads to the relatively compact form

$$\langle\mathbf{F}\rangle=\langle p_x\nabla E_x+p_y\nabla E_y+p_z\nabla E_z\rangle\ . \tag{53}$$

One further reduction is possible. The symmetry of the geometry dictates that the average force on a dipole is along the z axis joining the two particles. Therefore the expression for the average force finally simplifies to

$$\langle F\rangle=\langle F_z\rangle=\left\langle p_x\frac{\partial E_x}{\partial z}+p_y\frac{\partial E_y}{\partial z}+p_z\frac{\partial E_z}{\partial z}\right\rangle\ . \tag{54}$$

In determining the average force on, say, dipole B, we must keep in mind that the electric field at B consists of both the background ZPF field and the dipole field due to particle A, so that, for example, the first term in (54) becomes

$$\left\langle p_{Bx}\frac{\partial}{\partial z}E_x(\mathbf{r}_B,t)\right\rangle=\left\langle p_{Bx}\frac{\partial}{\partial z}E_{\mathrm{ZP}x}(\mathbf{r}_B,t)\right\rangle+\left\langle p_{Bx}\frac{\partial}{\partial z}E_{dAx}(\mathbf{r}_Bt)\right\rangle\ , \tag{55}$$

and so forth.

The mathematics of carrying out the averaging then proceeds term by term as in Ref. (27), using (as in previous sections) the $\frac{1}{2}$Re and complex-conjugate notation, including averaging over random phases. The result for the special case of interest here (radiation field only, binding and radiation-damping forces neglected) is, in terms of the potential U,[29]

$$U=-\frac{9}{4}\frac{\hbar c^3\Gamma^2}{\pi}\,\mathrm{Re}\int_0^{u_c}du\frac{e^{-2uR}}{R^2}, \quad u=-\frac{i\omega}{c}\ . \tag{56}$$

The only difference here as compared to the derivation in Ref. 27 (aside from the specialization to the radiation field term) is the use of a finite cutoff frequency.

This result is derived here and in Ref. 27 on the basis of point particles interacting with a classical zero-point field. For those who might be more familiar with standard quantum calculations, this result has also been obtained by Renne[30] from quantum-electrodynamic calculations using a nonrelativistic-quantum-oscillator model, and by Casimir and Polder[31] using fourth-order perturbation theory in quantum electrodynamics.

Equation (56) was derived for the case in which three degrees of freedom for particle motion are assumed. For the two-dimensional *Zitterbewegung* motion assumed in our case ($N=2$), geometrical considerations require that U be reduced by a factor $(N/3)^2=\frac{4}{9}$ (see Appendix A). With this taken into account the solution to (56) becomes

$$U=-\frac{\gamma}{2}\frac{1-\cos(2\mathcal{R})}{\mathcal{R}^3}=-\frac{\gamma}{\mathcal{R}}\left[\frac{\sin\mathcal{R}}{\mathcal{R}}\right]^2, \tag{57}$$

where

$$\gamma=\frac{\hbar\Gamma^2\omega_c^3}{\pi}, \quad \mathcal{R}=\frac{\omega_c R}{c}\ . \tag{58}$$

With the potential thus defined, the force is obtained from

$$\mathbf{F}=-\frac{\partial U}{\partial R}\ . \tag{59}$$

We see therefore that the potential has the desired $1/R$ dependence required for gravity, modulated by a fine-structure overlay of the form $[(\sin\mathcal{R})/\mathcal{R}]^2$ which has a spatial periodicity characteristic of the cutoff (Planck) frequency ($\sim 10^{-33}$ cm). If we extract the leading (nonoscillatory) term, we find for the potential and force

$$U=-\frac{\hbar c\Gamma^2\omega_c^2}{\pi R}+\cdots\ , \tag{60}$$

$$F=-\frac{\hbar c\Gamma^2\omega_c^2}{\pi R^2}+\cdots\ . \tag{61}$$

A careful examination of the details of averaging over the rapid spatial variation (see Appendix B) indicates that the particle experiences an average force $\langle F\rangle$ given by the leading term in (61). With Γ given by (23) and ω_c by (39), $\langle F\rangle$ can then be written in Newton's law form (with no adjustable parameters required),

$$\langle F\rangle=-\frac{Gm^2}{R^2}\ . \tag{62}$$

This derivation of Newtons's law, which expresses gravity as a van der Waals force, is thus the third and final result derived from the *Zitterbewegung* model.

VI. DISCUSSION

We begin our discussion by reiterating the logic flow of the approach pursued here. The basic thesis is the Sakharov proposal that gravity is not a separately existing fundamental force, but rather a residuum force derived from zero-point fluctuations of other fields in the manner of the Casimir and van der Waals forces. Particularizing this hypothesis to the ZPF of the vacuum electromagnetic field, we identify the gravitational force as the van der Waals force associated with the long-range radiation fields (as opposed to the usual shorter-range induction fields) generated by the *Zitterbewegung* particle (parton) motion response to the ZPF of the electromagnetic field. The steps in this identification are three.

First, particle mass is defined as the "internal" (that is, unobserved) kinetic energy of *Zitterbewegung* motion. Its value is set by a radiation damping constant Γ intrinsic to the particle, in conjunction with the value of the universal cutoff frequency ω_c. Second, the value of the cutoff frequency ω_c is determined by the equivalence principle. This principle sets a requirement that an expected additional contribution to the free-space Lorentz-frame ZPF spectrum, viewed from an accelerated frame, is to be equated to a similar contribution expected in an unaccelerated frame, but in a gravitational field (that is, near a particle performing *Zitterbewegung* motion, and thereby generating a mass term). Third, a straightforward calculation of the long-range van der Waals force associated with the radiation-field-correlated motions of such particles, whose parameters are determined by the above two steps, leads to Newton's law with no free parameters to be fixed.

With a detailed theory in hand, certain attributes of the gravitational interaction become explicable in fundamental terms. As mentioned earlier, the relative weakness of the gravitational force is due to the fact that the coupling constant determined by (40), $G=\pi c^5/\hbar\omega_c^2$, reflects as the inverse square the high value of the ZPF cutoff frequency. With regard to the attractive nature of the force, this is simply a reflection of a property typical of van der Waals forces in general. The fact that gravitational interaction is characterized by a unipolar (single-valued) "charge" (mass) can be traced to a (positive only) kinetic energy basis for the mass parameter.

The lack of shielding effects in gravity can also be comprehended on a rational basis. As understood here, this is a consequence of the fact that ZPF "noise" (quantum noise) in general cannot be shielded, a factor which in other contexts sets a lower limit on the detectability of electromagnetic signals. Specifically, in the case of ordinary electromagnetic shielding, macroscopic materials constitute dense boundaries that substantially alter field distributions, with shielding one consequence. In the gravitational case as modeled here, however, matter constitutes a dilute particle gas in an essentially high-frequency *Hohlraum*. As a result, particle-ZPF interactions have negligible effect on the overall field distribution (hence a lack of shielding), while nonetheless permitting particle-particle interactions that lead to an attractive potential.

In addition, implicit in the development pursued here are issues that extend beyond the gravitational interaction, such as mass renormalization and possible ZPF-induced contributions to the binding forces within the nucleus.[24,29] However, of the many inferences that can be drawn from this study, the most important is simply the fact that it is possible to carry through the basic Sakharov program, namely, to uncover a basis for gravity in the ZPF of other (nongravitational) fields. In particular, we have been able to explicate a first-order model[32] based on the ZPF of the vacuum electromagnetic field alone, once we take into account its effects on particle motion. The model thus details an electromagnetic basis for gravity. Assuming the model is a proper representation of the gravitational interaction, the "already unified" aspect of the model would seem to mitigate against canonical attempts at unification of gravity as a separate force, or quantization of gravity as a separate field, in favor of a viewpoint more aligned with that presented here.

It is therefore seen that a well-defined, precise quantitative argument can be made that gravity is a form of long-range van der Waals force associated with particle *Zitterbewegung* response to the zero-point fluctuations of the electromagnetic field. As such, the gravitational interaction takes its place alongside the short-range van der Waals forces and the Casimir force as related phenomena which emerge from the underlying dynamics of the interaction of particles with the zero-point fluctuations of the vacuum electromagnetic field.

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APPENDIX A: DIMENSIONAL REDUCTION FACTOR FOR COUPLING CONSTANT

As discussed in Sec. V, the attractive force between particles derives from coupling between ZPF-induced dipole motions of the particles involved. The nature of the coupling is simplified somewhat in that the coupling takes place only between corresponding components of the motions of the two particles; that is, between $\tilde{p}_{Ax}$ and $\tilde{p}_{Bx}$, $\tilde{p}_{Ay}$ and $\tilde{p}_{By}$, and $\tilde{p}_{Az}$ and $\tilde{p}_{Bz}$. This can be seen in the form taken by (43) and (44).

In the derivation which leads to expression (56) for the interaction potential U, reference to the original derivation in Ref. 27 shows that the potential is proportional to the quantity $(\eta_x^2+\eta_y^2+\eta_z^2)$, where η_x, etc., are as given here in (46) and (47). That derivation assumes full three-dimensional motion, with $\tilde{p}_{Ax}$ coupling to $\tilde{p}_{Bx}$, $\tilde{p}_{Ay}$ to $\tilde{p}_{By}$, and $\tilde{p}_{Az}$ to $\tilde{p}_{Bz}$.

For the two-dimensional *Zitterbewegung* motions posited here, however, we may for convenience analyze the couplings on the basis of assuming a random distribution of one-third each of x-oriented dipoles (y-z motion), y-oriented dipoles (x-z motion), and z-oriented dipoles (x-y motion). For two x-oriented dipoles, both y and z motions couple, yielding a contribution proportional to $\eta_y^2+\eta_z^2$. For an x-oriented dipole (y-z motion) coupling to a y-oriented dipole (x-z motion), only the z components couple, leading to a contribution proportional to η_z^2; and so forth. For the nine possible dipole-pair combinations, straightforward enumeration of the possibilities leads to an average coupling factor proportional to

$$(4\eta_x^2+4\eta_y^2+4\eta_z^2)/9=\tfrac{4}{9}(\eta_x^2+\eta_y^2+\eta_z^2)\ .$$

As a result, for the two-dimensional motions of interest here, a reduction factor of $\frac{4}{9}$ is to be applied to the value of the coupling constant obtained for the general three-dimensional case without constraints.

APPENDIX B: AVERAGE FORCE

In the *Zitterbewegung* model of gravity, the two-particle interaction potential based on the radiation field van der Waals effect is given by (57), repeated here,

$$U=-\frac{\gamma}{2}\frac{1-\cos(2\mathcal{R})}{\mathcal{R}^3}=-\frac{\gamma}{\mathcal{R}}\left[\frac{\sin\mathcal{R}}{\mathcal{R}}\right]^2 . \tag{B1}$$

As seen, this expression can be factored into two parts, one with a slow spatial variation, $1/\mathcal{R}$, and one with a rapid spatial variation (on the order of the Planck wavelength), $[(\sin\mathcal{R})/\mathcal{R}]^2$. Of interest in the gravitational interaction is not the rapidly varying component, but rather an average value, averaged over a distance large compared to the Planck wavelength. With the potential given by (B1), the (normalized) force is given by

$$\mathcal{F}=-\frac{\partial U}{\partial\mathcal{R}}=\frac{\gamma}{2}\frac{\partial}{\partial\mathcal{R}}\left[\frac{1-\cos(2\mathcal{R})}{\mathcal{R}^3}\right] . \tag{B2}$$

As particle separation changes by an amount $\Delta\mathcal{R}$, the corresponding change in potential is given by

$$\begin{aligned}\Delta U&=-\int_{\mathcal{R}_i}^{\mathcal{R}_i+\Delta\mathcal{R}}\mathcal{F}\,d\mathcal{R}\\&=-\frac{\gamma}{2}\int_{\mathcal{R}_i}^{\mathcal{R}_i+\Delta\mathcal{R}}\frac{\partial}{\partial\mathcal{R}}\left[\frac{1-\cos(2\mathcal{R})}{\mathcal{R}^3}\right]d\mathcal{R}\\&=-\frac{\gamma}{2}\left[\frac{1-\cos[2(\mathcal{R}_i+\Delta\mathcal{R})]}{(\mathcal{R}_i+\Delta\mathcal{R})^3}-\frac{1-\cos(2\mathcal{R}_i)}{\mathcal{R}_i^3}\right] .\end{aligned} \tag{B3}$$

Assuming integration over a full cycle of the Planck variation so that $\cos[2(\mathcal{R}_i+\Delta\mathcal{R}]=\cos(2\mathcal{R}_i)$, and recognizing that $\Delta\mathcal{R}\ll\mathcal{R}_i$ so that $(\mathcal{R}_i+\Delta\mathcal{R})^3\approx\mathcal{R}_i^3+3\mathcal{R}_i^2\Delta\mathcal{R}$, we find that (B3) simplifies to

$$\Delta U=\frac{3\gamma\,\Delta\mathcal{R}}{2}\left[\frac{1-\cos(2\mathcal{R}_i)}{\mathcal{R}_i^4}\right] . \tag{B4}$$

The change in potential, integrated over a cycle, is seen from (B4) to be sensitive to where in the cycle, $\theta_i = 2\mathcal{R}_i$, the integration was begun. The *average* change in potential is therefore determined by averaging over the range of possible initial starting points within the cycle, namely, $\pi n \le \mathcal{R}_i \le \pi(n+1)$, where n is an integer $n \gg 1$.

By reference to standard tables of integrals[33] we find, using (B4),

$$\langle \Delta U \rangle = \frac{1}{\pi}\int_{\pi n}^{\pi(n+1)} \Delta U \, d\mathcal{R}_i$$
$$= -\frac{\gamma\,\Delta\mathcal{R}}{2\pi}\left[\frac{1}{\mathcal{R}_i^3}\right]_{\pi n}^{\pi(n+1)} - \frac{2\gamma\,\Delta\mathcal{R}}{\pi}\left[-\frac{2\cos(2\mathcal{R}_i)}{(2\mathcal{R}_i)^3} + \frac{\sin(2\mathcal{R}_i)}{(2\mathcal{R}_i)^2} + \frac{\cos(2\mathcal{R}_i)}{2\mathcal{R}_i} + \left[2\mathcal{R}_i - \frac{(2\mathcal{R}_i)^3}{3\times 3!} + \frac{(2\mathcal{R}_i)^5}{5\times 5!} - \cdots\right]\right]_{\pi n}^{\pi(n+1)} . \quad \text{(B5)}$$

Substitution of the limits of integration, with the recognition that $n \gg 1$ implies that $(n+1)^p \approx n^p + pn^{p-1}$, then leads to

$$\langle \Delta U \rangle = \frac{\gamma\,\Delta\mathcal{R}}{(\pi n)^2} - \frac{2\gamma\,\Delta\mathcal{R}}{\pi n}\left[2\pi n - \frac{(2\pi n)^3}{3!} + \frac{(2\pi n)^5}{5!} - \cdots\right] . \quad \text{(B6)}$$

But the term in large parentheses is recognized to be $\sin(2\pi n)=0$, so that $\langle \Delta U \rangle$ becomes

$$\langle \Delta U \rangle = \frac{\gamma\,\Delta\mathcal{R}}{\mathcal{R}^2} , \quad \text{(B7)}$$

from which the average force can be calculated as

$$\langle \mathcal{F} \rangle = -\frac{\langle \Delta U \rangle}{\Delta\mathcal{R}} = -\frac{\gamma}{\mathcal{R}^2} . \quad \text{(B8)}$$

The actual (unnormalized) force, $F = -\partial U/\partial R$, is recovered from the above with the aid of (58), yielding

$$\langle F \rangle = \frac{\omega_c}{c}\langle \mathcal{F} \rangle = -\frac{\hbar c \Gamma^2 \omega_c^2}{\pi R^2} . \quad \text{(B9)}$$

[1] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), pp. 417–428.

[2] C. W. Misner, K. S. Thorne, and J. A. Wheeler, Ref. 1, pp. 426–428.

[3] A. D. Sakharov, Dokl. Akad. Nauk SSSR [Sov. Phys.—Dokl. **12**, 1040 (1968)].

[4] These include the following: Sakharov's own conjecture that the Lagrange function of the gravitational field is generated by vacuum polarization effects due to fermions [A. D. Sakharov, Theor. Math. Phys. **23**, 435 (1975)]; the generation of gravity as a collective excitation of fermion-antifermion pairs [K. Akama, Y. Chikashige, T. Matsuki, and H. Terazawa, Prog. Theor. Phys. **60**, 868 (1978)]; proof that curvature can arise from the quantum fluctuations of pure gauge fields [B. Hasslacher and E. Mottolo, Phys. Lett. **95B**, 237 (1980)]; the generation of gravity as a symmetry-breaking effect in quantum field theory in which a dynamical scale-invariance breaking is postulated to take place at energies near the Planck mass [A. Zee, Phys. Rev. Lett. **42**, 417 (1979); Phys. Rev. D **23**, 858 (1981)]; "pregeometric" models in which the Einstein action and metric are generated from quantum fluctuations of matter fields [D. Amati and G. Veneziano, Phys. Lett. **105B**, 358 (1981); S. Yoshimoto, Prog. Theor. Phys. **78**, 435 (1987)].

[5] S. Adler, Rev. Mod. Phys. **54**, 729 (1982). In this review particular emphasis is placed on the case of renormalizable field theories with dynamical scale-invariance breaking, in which the induced gravitational effective action is finite and calculable.

[6] In the terminology of gravity as a symmetry-breaking effect, equations of this form are an immediate consequence of a postulated dynamical scale-invariance breaking that is assumed to take place near the Planck mass energy. For details see A. Zee, Ref. 4.

[7] In the sense that gravity as a non-Minkowskian curvature effect can be treated in first approximation as a Newtonian force in flat space-time, so can it be treated as a van der Waals–type force in flat space-time, and it is this basic concept that is addressed here. Although beyond the intended scope of this paper, an intimate connection between flat-space-time van der Waals and Casimir effects and vacuum curvature effects can be traced as a problem in the restructuring of vacuum energy, as in B. S. DeWitt, Phys. Rep. **19**, 295 (1975), especially pp. 303–308. Thus the connection between a flat-space-time approach and gravity as a curvature effect can be established as in general relativity generally.

[8] Although the approach taken here is to assume the reality of the ZPF of the electromagnetic field, for completeness we note that an alternative viewpoint posits that the results of field-particle interactions traditionally attributed to ZPF can also be expressed in terms of the radiation reaction of the particles involved, without explicit reference to the ZPF. [See P. W. Milonni, in *Foundations of Radiation Theory and Quantum Electrodynamics*, edited by A. O. Barut (Plenum, New York, 1980); Phys. Rev. A **25**, 1315 (1982).] The interrelationship between these two approaches (ZPF, radiation reaction) can be shown to be complementary on the basis of an

underlying fluctuation-dissipation theorem.

[9]For further discussion of the SED approach, see, for example, H. E. Puthoff, Phys. Rev. D **35**, 3266 (1987), and references therein. Specifically, for a detailed description of the correspondence between this approach and QED treatments of linear dipoles plus radiation field systems, see P. W. Milonni, Phys. Rep. **25**, 1 (1976), especially pp. 71–78. See also reviews of SED by T. H. Boyer, in *Foundations of Radiation Theory and Quantum Electrodynamics*, edited by A. O. Barut (Plenum, New York, 1980); L. de la Pena, in *Proceedings of the Latin American School of Physics, Cali, Columbia, 1982*, edited by B. Gomez *et al.* (World Scientific, Singapore, 1983).

[10]E. M. Lifshitz, Zh. Eksp. Teor. Fiz. **29**, 94 (1955) [Sov. Phys.—JETP **2**, 73 (1956)].

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[12]T. H. Boyer, Phys. Rev. D **11**, 790 (1975).

[13]T. H. Boyer, Phys. Rev. A **5**, 1799 (1972); **6**, 314 (1972); **7**, 1832 (1973); **9**, 2078 (1975); **11**, 1650 (1975).

[14]T. H. Boyer, Phys. Rev. **182**, 1374 (1969).

[15]T. H. Boyer, Phys. Rev. D **21**, 2137 (1980). See also J. S. Kim, K. S. Soh, S. K. Kim, and J. H. Yee, Phys. Rev. D **36**, 3700 (1987).

[16]P. C. W. Davies, J. Phys. A **8**, 609 (1975).

[17]W. G. Unruh, Phys. Rev. D **14**, 870 (1976).

[18]In a later paper [T. H. Boyer, Phys. Rev. D **29**, 1089 (1984)] Boyer shows that under the usual low-frequency peaked-resonance approximation the additional density-of-states factor cancels out, with the result that to this order of approximation gravitational effects are neglected. As will be shown in Sec. III, however, it is in the emergence of high-frequency behavior (where the additional density-of-states factor remains operative) that gravitational effects have their origin.

[19]K. Huang, Am. J. Phys. **20**, 479 (1952).

[20]A. Rueda, Phys. Rev. A **23**, 2020 (1981).

[21]The possible existence of a cutoff in quantum theory is recognized to introduce a non-Lorentz-invariant factor, in that detection of a Doppler-shifted cutoff frequency by a moving detector could in principle reveal absolute motion. As pointed out in the literature, however [for example, by M. A. Shupe, Am. J. Phys. **53**, 122 (1985)], as long as the cutoff frequency is beyond detectability (as is the Planck frequency in this case) there is no measurable consequence expected of such a breakdown of Lorentz invariance at this limit of present physical theory, either now or in the foreseeable future.

[22]See, for example, W. Heitler, *The Quantum Theory of Radiation*, 3rd ed. (Oxford University Press, London, 1954), pp. 293 ff.

[23]D. Bohm, *The Special Theory of Relativity* (Benjamin-Cummings, Reading, MA, 1965), Chap. 19.

[24]The particle mass given by (23), $m=\Gamma c^3/G$, corresponds to a renormalized or "dressed" mass, and it is this mass (of dynamical origin) that appears in the field-generation effects and interaction potential corresponding to gravity. If one wished to do so, the derivation that led to (23) would permit one to extract an unobserved harmonic-oscillator "bare" mass m_0 which appears in the definition of Γ preceding (8). This unrenormalized mass is given by the large value $m_0 \sim (m_P^2/m)$, where $m_P=\sqrt{\hbar c/G}$ is the Planck mass. Although unobserved experimentally, it is this unrenormalized mass m_0 which could be said to interact at the level of the ZPF. Viewed in the usual QED terms, its value is so high, and its Compton wavelength so small, as to justify its treatment as a point particle (as we have done), even at wavelengths corresponding to the ZPF (Planck) cutoff frequency. Lacking any apparent dynamic origin or interaction effects at an observable level, however, the concept of a bare mass can be seen to be simply an inference derived from the particle parameter Γ, which appears to be the operative parameter of significance. Therefore questions as to why the bare mass does not generate a large gravitational field, which can be taken to be related to the well-known field-theoretic conundrum as to why the vacuum zero-point energy does not do likewise, find a simple solution in the Sakharov approach as pursued here. In the dynamics of the model the only gravitational fields found to emerge from the analysis are those that are in fact observed experimentally, and this is one of the strengths of the approach.

[25]See, for example, J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941), p. 435.

[26]See T. H. Boyer, Ref. 12, Appendix B.

[27]T. H. Boyer, Phys. Rev. A **7**, 1832 (1973).

[28]T. H. Boyer, Ref. 27, Eqs. (56) and (57).

[29]The complete expression, including both radiation and induction field terms, is given by (92) in Ref. 27. It is (in our units, and for $\omega_0=0$)

$$U=-\frac{9}{4}\frac{\hbar c^3\Gamma^2}{\pi}\,\mathrm{Re}\int_0^{u_c} du\,\frac{e^{-2uR}}{R^2}\left[1+\frac{2}{uR}+\frac{5}{(uR)^2}+\frac{6}{(uR)^3}+\frac{3}{(uR)^4}\right].$$

This constitutes the general expression for the retarded van der Waals forces at all distances. While the first (radiation field) term in large parentheses is found here to account for the long-range gravitational interaction, the remaining short-range (induction field) terms have yet to be investigated with regard to their contribution to binding at the nuclear (parton and nucleon) level.

[30]M. J. Renne, Physica **53**, 193 (1971).

[31]H. B. G. Casimir and D. Polder, Phys. Rev. **73**, 360 (1948).

[32]Further development of the model could include, for example, extending parton *Zitterbewegung* to the relativistic case.

[33]See, for example, H. B. Dwight, *Tables of Integrals and Other Mathematical Data* (Macmillan, New York, 1961), especially Eqs. 441.14 and 431.11.